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Interpolating Equations Between Two Limiting Cases For The Heat Transfer Coefficient

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Simple analytical solutions are available for the heat transfer coefficient for laminar forced or free convection along a vertical plate when Prandtl number is sufficiently large. When, however, both forced and free convection are superimposed, no simple solution can be obtained. Equations expressing the Nusselt number Nu_x in terms of the Nusselt number $Nu_{x,F}$ for forced convection and the Nusselt number $Nu_{x,N}$ for natural convection have been suggested for laminar, assisting, forced, and free convection. They have the form

$$Nu_x = Nu^n_{x,F} + Nu^n_{x,N} \tag{1}$$

The exponent n was chosen between 2 and 4. Churchill (1977) has reviewed recently this kind of expressions and shows that $n = 3$ gives the best representation of the data. Some new arguments are adduced in what follows supporting Churchill's choice. An approach is used (Ruckenstein, 1962) which can interpolate between two extreme cases when each extreme can be treated in terms of a boundary-layer approximation. This kind of interpolation was used before (Ruckenstein, 1962) to obtain the heat transfer coefficient during inpulsive heating of a liquid in steady laminar flow along a plate, as well as to obtain an equation for the mass transfer coefficient between a spherical drop and the continuous phase covering as limiting cases the Hadamard and Stokes flows (Ruckenstein, 1964).

For large Prandtl numbers, only the region very near the wall contributes appreciably to the rate of heat transfer. Since in this region the inertial (nonlinear) terms are small compared to the other terms, the velocity components u and v will be written as the sum of two components

$$u = u_1 + u_2 \quad \text{and} \quad v = v_1 + v_2 \tag{2}$$

where u_1 and v_1 are the velocity components for pure forced convection and u_2 and v_2 are those for free convection. Consequently, for large Prandtl numbers, the temperature field satisfies the equation

$$(u_1 + u_2)\frac{\partial T}{\partial x} + (v_1 + v_2)\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \tag{3a}$$

while the velocities u_2 and v_2, due to the free convection, satisfy the equations

$$u_2\frac{\partial u_2}{\partial x} + v_2\frac{\partial u_2}{\partial y} = \nu\frac{\partial^2 u_2}{\partial y^2} + \beta g(T - T_\infty) \tag{3b}$$

and

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \tag{3c}$$

Since the velocities u_1 and v_1 are known, the decomposition (2) has partially decoupled free and forced convection. Equations (3) are still difficult to solve, and therefore a complete decoupling is carried out by computing u_2 and v_2 with the temperature profile for pure free convection [hence solving Equations (3*b*) and (3*c*) together with $u_2(\partial T/\partial x) + v_2(\partial T/\partial y) = a(\partial^2 T/\partial y^2)$ instead of with Equation (3*a*)]. It is difficult to specify exactly how good is this approximation. An upper bound of the error can be obtained as follows. First one may notice that the equations represent well the two limiting cases $Re_x \to 0$ and $Ra_x \to 0$. The difficult region is that in which both effects are equally important. This happens when u_1 and u_2 are of the same order. Using a velocity half as large as

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the true one to evaluate $T - T_\infty$, a maximum error of about $(2^{1/3} - 1) \times 100\%$ results. Hence, the velocity u_2 is obtained with a maximum error of about 40%. The maximum error in the final $T - T_\infty$ obtained solving Equation (3a) is, however, much smaller because adding u_1 to u_2, the error in velocity is reduced to about 20%, and this error is further reduced in the temperature profile to about 10%. Very near the solid wall we use for the velocity profile the first term in the Blasius expression for forced convection and Levich's expression for free convection (Levich, 1962):

$$u_1 = \frac{1.33}{4}\frac{u_o^{3/2}y}{\nu^{1/2}x^{1/2}}, \quad v_1 = \frac{1.33}{16}\frac{u_o^{3/2}y^2}{\nu^{1/2}x^{3/2}} \qquad (4a)$$

and

$$u_2 = 1.92\nu\, Pr^{-1/4}\left(\frac{g\beta\Delta T}{4\nu^2}\right)^{3/4} x^{1/4}y$$

$$v_2 = -\,0.24\, Pr^{-1/4}\left(\frac{g\beta\Delta T}{4\nu^2}\right)^{3/4} x^{-3/4}y^2 \qquad (4b)$$

With the mentioned approximations, the temperature field near the solid surface satisfies the equation

$$(u_1 + u_2)\frac{\partial T}{\partial x} + (v_1 + v_2)\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \qquad (5)$$

where u_1, u_2, v_1, and v_2 are given by Equations (4). Let us denote by $\delta = k/h_x$, the thickness of the thermal boundary layer. Evaluating each of the terms of Equation (5) by replacing ∂T by ΔT, ∂x by x, ∂y by δ, and y by δ, one obtains

$$(u_1 + u_2)\frac{\partial T}{\partial x} \sim \left[\frac{u_o^{3/2}\delta}{\nu^{1/2}x^{1/2}} + \psi\nu Pr^{-1/4}\left(\frac{g\beta\Delta T}{\nu^2}\right)^{3/4} x^{1/4}\,\delta\right]\frac{\Delta T}{x} \qquad (6a)$$

$$(v_1 + v_2)\frac{\partial T}{\partial y} \sim \left[\frac{u_o^{3/2}\delta}{\nu^{1/2}x^{1/2}} + \psi'\nu Pr^{-1/4}\left(\frac{g\beta\Delta T}{\nu^2}\right)^{3/4} x^{1/4}\,\delta\right]\frac{\Delta T}{x} \qquad (6b)$$

and

$$\frac{\partial^2 T}{\partial y^2} \sim \frac{\Delta T}{\delta^2} \qquad (6c)$$

The symbol $\sim$ has the meaning of the order of.

Replacing each of the terms of Equation (5) with the corresponding expression (6) multiplied by a constant, one obtains

$$A\frac{u_o^{3/2}\delta}{\nu^{1/2}x^{3/2}} + B\nu Pr^{-1/4}\left(\frac{g\beta\Delta T}{\nu^2 x}\right)^{3/4}\delta = \frac{a}{\delta^2} \qquad (7)$$

where A and B are constants. Because

$$Nu_x \equiv \frac{x}{\delta} \qquad (8)$$

Equation (7) can be rewritten as

$$A\, Re_x^{3/2}\, Pr + B\, Ra_x^{3/4} = Nu_x^3 \qquad (9)$$

where

$$Re_x \equiv \frac{u_o x}{\nu}, \quad Pr \equiv \frac{\nu}{a} \quad \text{and} \quad Ra_x \equiv \frac{g\beta\Delta T x^3}{\nu^2} Pr \qquad (10)$$

The constants A and B can be obtained observing that for large Prandtl numbers and uniform wall temperature

$$Nu_x = 0.339\, Re_x^{1/2}\, Pr^{1/3} \qquad (11)$$

for pure convection and

$$Nu_x = 0.503\, Ra_x^{1/4}$$

for pure free convection. When instead of the temperature the heat flux is uniform

$$Nu_x = 0.464\, Re_x^{1/2}\, Pr^{1/3} \qquad (12)$$

for pure forced convection and

$$Nu_x = 0.563\, Ra_x^{1/4}$$

for pure free convection. The final result is the equation

$$Nu_x^3 = Nu_{x,F}^3 + Nu_{x,N}^3 \qquad (13)$$

suggested by Churchill (1977). Equation (13) was established using an approximate interpolation procedure and a reasonable additivity assumption concerning the velocity field. Let us observe in passing that Equation (5) can be solved exactly using either the similarity transformation $\eta = y/X(x)$ or the kind of transformations suggested by Ruckenstein (1971). The result differs, however, by at most, 8% from the one given by Equation (13).

NOTATION

- a = thermal diffusivity
- A, B = numerical constants
- g = acceleration of gravity
- h_x = local heat transfer coefficient
- k = thermal conductivity
- Nu_x = Nusselt number $\equiv h_x x/k$
- $Nu_{x,F}$ = Nusselt number for forced convection
- $Nu_{x,N}$ = Nusselt number for natural convection
- Pr = Prandtl number $\equiv \nu/a$
- Ra_x = local Rayleigh number $\equiv (g\beta\Delta T x^3)/(\nu^2)Pr$
- Re_x = local Reynolds number $\equiv u_o x/\nu$
- T = temperature
- T_w = wall temperature
- T_∞ = temperature at large distances from the wall
- ΔT = $T_w - T_\infty$
- u = x component of velocity
- u_o = velocity at large distances from the wall
- v = y component of velocity
- x = distance up the plate
- y = distance to the plate
- β = volumetric coefficient of expansion
- δ = thickness of the thermal boundary layer
- ψ, ψ' = numerical constants
- ν = kinematic viscosity

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